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## Fractional Cyclical Structures and Business Cycles in the Specification of the US Real Output

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### **Abstract**

*The issue in this paper is to analyse the business cycle frequencies in the US real output. However, instead of using classical approaches based on linear and non-linear models, we use a specification of fractional cyclical integration, which is based on Gegenbauer processes. We apply a procedure that permits us to test roots with integer and fractional orders of integration at fixed frequencies over time and thus, it permits us to approximate the length of the cycles. The results, based on the first differenced data, show that the cycles have a duration of about four years and a half, with an order of integration higher than 0 but smaller than 0.5, being thus stationary but with a component of long memory behaviour. Comparing this model with those based on ARIMA (and ARFIMA) models, we show via simulations that the fractional cyclical structure can better describe the business cycle features of the data.*

**Keywords:** *Gegenbauer processes; Business cycles; Long memory; US output.*

**JEL classification:** *C22.*

### **1. Introduction**

The existence of cycles in macroeconomic time series is a well-known stylised fact. However, its appropriate way of modelling is a matter that still remains controversial. With the development of the National Bureau of Economic Research (NBER)'s project of "Measurement without Theory" and the first extensive study of Burns and Mitchell (1946) on the American Economy, business cycles and their features have constituted a direct object of empirical analysis. Numerous studies have tried to describe them and to consider their stability over time. Romer (1986, 1994), Diebold and Rudebush (1992) and Watson (1994) have, for example, explored data to know if fluctuations have been smoother (lower amplitude and longer duration) after the Second World War. Also, Neftci (1984), Hamilton (1989), Beaudry and

Koop (1993) investigated new business cycles features<sup>1</sup>, showing that cycles exhibit an asymmetry in their phases: recessions being deeper and shorter than expansions. Other authors, such as Candelon and Henin (1995) characterised the distributions of these features via bootstrapped simulation of simple linear (ARIMA) models for GDP. They could then locate the observed features of the last cycle and conclude that they are rather normal. A step further, Hess and Iwata (1997) used them as benchmarks to gauge the adequacy of macroeconomic stochastic time series models. They replicate via Monte-Carlo simulations different models for GDP. Then, they build for each model the distribution of the business cycle features and compare them to the historical business cycle characteristics. The best model is then selected as the one, which best replicates the historical features. Three types of linear models, namely, integrating a stochastic trend (ARIMA), a deterministic trend and a segmented trend (as in Perron, 1989) as well as several non-linear ones (SETAR, Markov-Switching and Beaudry and Koop, 1993) are considered. They conclude that complex non-linear or linear models do not better replicate business cycle features than a simple linear ARIMA(1,1,0). Such a conclusion appears to be rather destructive for recent attempts, which have tried to better model GDP. Pesaran and Potter (1997) consider a non-linear time series model to examine the non-linearity in US output, while Candelon and Gil-Alana (2004) use fractionally ARIMA (ARFIMA) models to characterise the business cycle components of the US and the UK GDP.

Another issue is to estimate the length of the cycles. In modern business cycle research, both empirical and theoretical macroeconomic and model-generated time series are detrended before analysis of the business cycle component. A vast majority of researchers use the Hodrick-Prescott (1997) filter (HP-filter) or Baxter and King's (1999) band-pass filter, and most authors conclude that business cycles have a duration of about six years. The HP filter has been interpreted as an approximation to an ideal high pass filter, eliminating frequencies of 32 quarters or greater. (See, e.g., Prescott, 1986, and King and Rebelo, 1993). Researchers relying on other methods often share this view about the duration of the business cycle component. Baxter and King (1999) construct a band-pass filter designed to extract cycles with duration between 1.5 and 8 years. Englund et al. (1992) and Hassler et al. (1994) use a band-pass filter in the frequency domain to extract cycles with duration between 3 and 8 years. Similar conclusions are obtained in Canova (1998), Burnside (1998), King and Rebelo (1999) and others.

In this article, we model the business cycle characteristics of the US real GDP from a different time series perspective, and consider the cycle as an

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<sup>1</sup> These features integrate the third moment of the cycle as well as the conditional asymmetry in mean.

additional component to the trend and to the seasonal structure of the series. Similarly to these other two components, deterministic cycles (based in this context, on trigonometric functions of time) were shown to be inappropriate for modelling most macroeconomic time series. Harvey (1985) and others proposed stochastic cycles, and Gray et al. (1989, 1994) generalised them to allow for long memory. In particular, these authors considered processes like:

$$(1 - 2\mu L + L^2)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (1)$$

where  $L$  is the lag-operator ( $Lx_t = x_{t-1}$ ),  $d$  can be any real number, and where  $u_t$  is an  $I(0)$  process, defined, in the context of the present paper, as a covariance stationary process with spectral density function, which is bounded and bounded away from zero at any frequency on the spectrum. Clearly, when  $d = 0$  in (1),  $x_t = u_t$ , and a “weakly autocorrelated”  $x_t$  is allowed for, as opposed to the case of  $d > 0$  when the process is said to be “strongly autocorrelated” or also called “strongly dependent”, so-named because of the strong association (in the cyclical part) between observations widely separated in time. Gray et al. (1989) showed that  $x_t$  in (1) is stationary if  $|\mu| < 1$  and  $d < 0.50$  or if  $|\mu| = 1$  and  $d < 0.25$ . They also showed that the polynomial in (1) can be expressed in terms of the Gegenbauer polynomial  $C_{j,d}(\mu)$ , such that for all  $d \neq 0$ ,

$$(1 - 2\mu L + L^2)^{-d} = \sum_{j=0}^{\infty} C_{j,d}(\mu) L^j, \quad (2)$$

where

$$C_{j,d}(\mu) = \sum_{k=0}^{[j/2]} \frac{(-1)^k (d)_{j-k} (2\mu)^{j-2k}}{k!(j-2k)!}; \quad (d)_j = \frac{\Gamma(d+j)}{\Gamma(d)},$$

$\Gamma(x)$  is the Gamma function, and a truncation will be required in (2) to make (1) operational. Thus, the process in (1) becomes:

$$x_t = \sum_{j=0}^{t-1} C_{j,d}(\mu) u_{t-j}, \quad t = 1, 2, \dots, \quad (3)$$

and when  $d = 1$ , we have

$$x_t = 2\mu x_{t-1} - x_{t-2} + u_t, \quad t = 1, 2, \dots, \quad (4)$$

which is a cyclical  $I(1)$  process with the periodicity determined by  $\mu$ . Tests of (4) based on autoregressive (AR) alternatives were proposed amongst others by Ahtola and Tiao (1987). Their tests are embedded in an  $AR(2)$  process of form:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + u_t, \quad (5)$$

which, under the null hypothesis:

$$H_0: |\phi_1| < 2 \text{ and } \phi_2 = -1, \quad (6)$$

becomes the cyclical I(1) model (4). Unit root cycles were also examined by Chan and Wei (1988), Chung (1996) and Gregoir (1999a,b), who derive the limiting distribution of least squares estimates of AR processes with complex-conjugate unit roots, with inference based on parametric estimates.

In this article we use the fractional structure (1) for testing cyclical roots with integer and fractional orders of integration in raw time series. The use of this parametric approach to investigate the long run behaviour of time series consists of testing a parametric model for the series and relying on the long run implications of the estimated model. The primary advantage is the precision gained by focusing on the information in the series through the parameter estimates. A drawback is that the parameter estimates are sensitive to the class of models considered and may be misleading because of misspecification. However, the possibility of misspecification with parametric models can never be settled conclusively, and the problem can be addressed by considering a large class of models. For this purpose, we employ a version of the tests of Robinson (1994) that permits us to test long memory cyclical models. An advantage of this procedure is that is based on the Lagrange Multiplier (LM) principle and therefore we do not have to estimate any long memory fractional (cyclical) parameter. These tests are briefly explained in Section 2. Section 3 applies them to the quarterly structure of the US real output. In Section 4, we make some simulations comparing the business cycle characteristics of several models, while Section 5 contains some concluding comments.

## 2. The testing procedure

Following Bhargava (1986), Schmidt and Phillips (1992) and others on parameterization of unit-root models, Robinson (1994) considers the regression model:

$$y_t = \beta' z_t + x_t \quad t = 1, 2, \dots, \quad (7)$$

where  $y_t$  is a given raw time series;  $z_t$  is a  $(k \times 1)$  vector of exogenous variables;  $\beta$  is a  $(k \times 1)$  vector of unknown parameters; and the regression errors  $x_t$  are such that:

$$\rho(L; \theta) x_t = u_t \quad t = 1, 2, \dots, \quad (8)$$

where  $\rho$  is a given function, which depends on  $L$  and the  $(p \times 1)$  parameter vector  $\theta$ , adopting the form:

$$\rho(L; \theta) = (1-L)^{d_1+\theta_1} (1-L^4)^{d_2+\theta_2} \prod_{j=3}^p (1-2 \cos w L + L^2)^{d_j+\theta_j} \quad (9)$$

for real given numbers  $d_1, d_2, \dots, d_p$ , and integer  $p$ , and where  $u_t$  is an  $I(0)$  process<sup>2</sup>. Under the null hypothesis:

$$H_0: \theta = 0, \quad (10)$$

(9) becomes:

$$\rho(L; \theta = 0) = \rho(L) = (1 - L)^{d_1} (1 - L^4)^{d_2} \prod_{j=3}^p (1 - 2 \cos w L + L^2)^{d_j}. \quad (11)$$

This is a very general specification that permits us to consider different models under the null. For example, if  $d_1 = 1$  and  $d_j = 0$  for  $j \geq 2$ , we have the classical unit-root model (Dickey and Fuller, 1979; Phillips and Perron, 1988; or the alternative in Kwiatkowski et al., 1992, etc.) and, if  $d_1$  is a real value, the fractional models examined in Diebold and Rudebusch (1989), Baillie (1996) and others. Similarly, imposing  $d_2 = 1$  and  $d_j = 0$  for  $j \neq 2$ , we have the seasonal unit root model (Dickey, Hasza and Fuller, 1984, Hylleberg et al., 1990, etc.) and if  $d_2$  is real, the seasonal fractional model analysed in Porter-Hudak (1990) and Gil-Alana (2002). Finally, if  $d_1 = d_2 = 0$  and  $d_3 = 1$ , we have the unit root cycles of Ahtola and Tiao (1987); if  $d_3$  is real, the model in Gray et al. (1989) and, if  $j > 3$ , the  $k$ -factor Gegenbauer processes studied in Ferrara and Guégan (2001).

In this article, we are concerned with the cyclical structure of the series and thus, we take  $d_j = 0$  for  $j \neq 3$  and  $d_3 = d$ . In such a situation (9) becomes:

$$\rho(L; \theta) = (1 - 2 \cos w L + L^2)^{d+\theta} \quad (12)$$

and similarly (11),  $\rho(L) = (1 - 2 \cos w L + L^2)^d$ . Plugging (12) in (7) and (8),  $y_t$  follows, under the null, a cyclical  $I(d)$  model of the form advocated by Gray (1989, 1994), and imposing  $d = 1$ ,  $y_t$  appears as the unit root cycles proposed by Ahtola and Tiao (1987) and others.

We next describe the test statistic. We observe  $\{(y_t, z_t), t = 1, 2, \dots, n\}$ , and suppose that the  $I(0)$   $u_t$  in (8) have spectral density given by:

$$f(\lambda; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau), \quad -\pi < \lambda \leq \pi,$$

where the scalar  $\sigma^2$  is known and  $g$  is a function of known form, which depends on frequency  $\lambda$  and the unknown  $(qx1)$  vector  $\tau$ . Based on  $H_0$  (10), the residuals in (7); (8) and (12) are

$$\hat{u}_t = (1 - 2 \cos w L + L^2)^d y_t - \hat{\beta}' s_t, \quad (13)$$

<sup>2</sup> In Robinson (1994), the function  $\rho(L; \theta)$  is slightly different. However, we have preferred to use this specification to stress the different components of the series, i.e., the trend, the seasonal and the cycles.

$$\text{where } \hat{\beta} = \left( \sum_{t=1}^n s_t s_t' \right)^{-1} \sum_{t=1}^n s_t (1 - 2 \cos wL + L^2)^d y_t,$$

$$\text{with } s_t = (1 - 2 \cos wL + L^2)^d z_t.$$

Unless  $g$  is a completely known function (e.g.,  $g \equiv 1$ , as when  $u_t$  is white noise), we have to estimate the nuisance parameter  $\tau$ , for example by  $\hat{\tau} = \arg \min_{\tau \in T^*} \sigma^2(\tau)$ , where  $T^*$  is a suitable subset of  $\mathbb{R}^q$  Euclidean space, and

$$\sigma^2(\tau) = \frac{2\pi}{n} \sum_{s=1}^{n-1} g(\lambda_s; \tau)^{-1} I_{\hat{u}}(\lambda_s), \quad \text{with}$$

$$I_{\hat{u}}(\lambda_s) = \left| (2\pi n)^{-1/2} \sum_{t=1}^n \hat{u}_t e^{i\lambda_s t} \right|^2; \quad \lambda_s = \frac{2\pi s}{n}.$$

The test statistic, which is derived via Lagrange Multiplier (LM) principle, takes the form:

$$\hat{R} = \hat{r}^2; \quad \hat{r} = \left( \frac{T}{\hat{A}} \right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \quad (14)$$

where

$$\hat{a} = \frac{-2\pi}{n} \sum_{s=1}^* \psi(\lambda_s) g(\lambda_s; \hat{\tau})^{-1} I_{\hat{u}}(\lambda_s); \quad \hat{\sigma}^2(\tau) = \sigma^2(\hat{\tau});$$

$$\hat{A} = \frac{2}{n} \left( \sum_{s=1}^* \psi(\lambda_s)^2 - \sum_{s=1}^* \psi(\lambda_s) \hat{\varepsilon}(\lambda_s)' \left( \sum_{s=1}^* \hat{\varepsilon}(\lambda_s) \hat{\varepsilon}(\lambda_s)' \right)^{-1} \sum_{s=1}^* \hat{\varepsilon}(\lambda_s) \psi(\lambda_s) \right),$$

$$\psi(\lambda_s) = \log |2 (\cos \lambda_s - \cos w)|; \quad \hat{\varepsilon}(\lambda_s) = \frac{\partial}{\partial \tau} \log g(\lambda_s; \hat{\tau}),$$

and the sum over  $*$  in the above expressions refers to all the discrete frequencies  $\lambda_s$  except the one with a pole in  $\psi(\lambda)$ .

Robinson (1994) showed that under certain very mild regularity conditions<sup>3</sup>,

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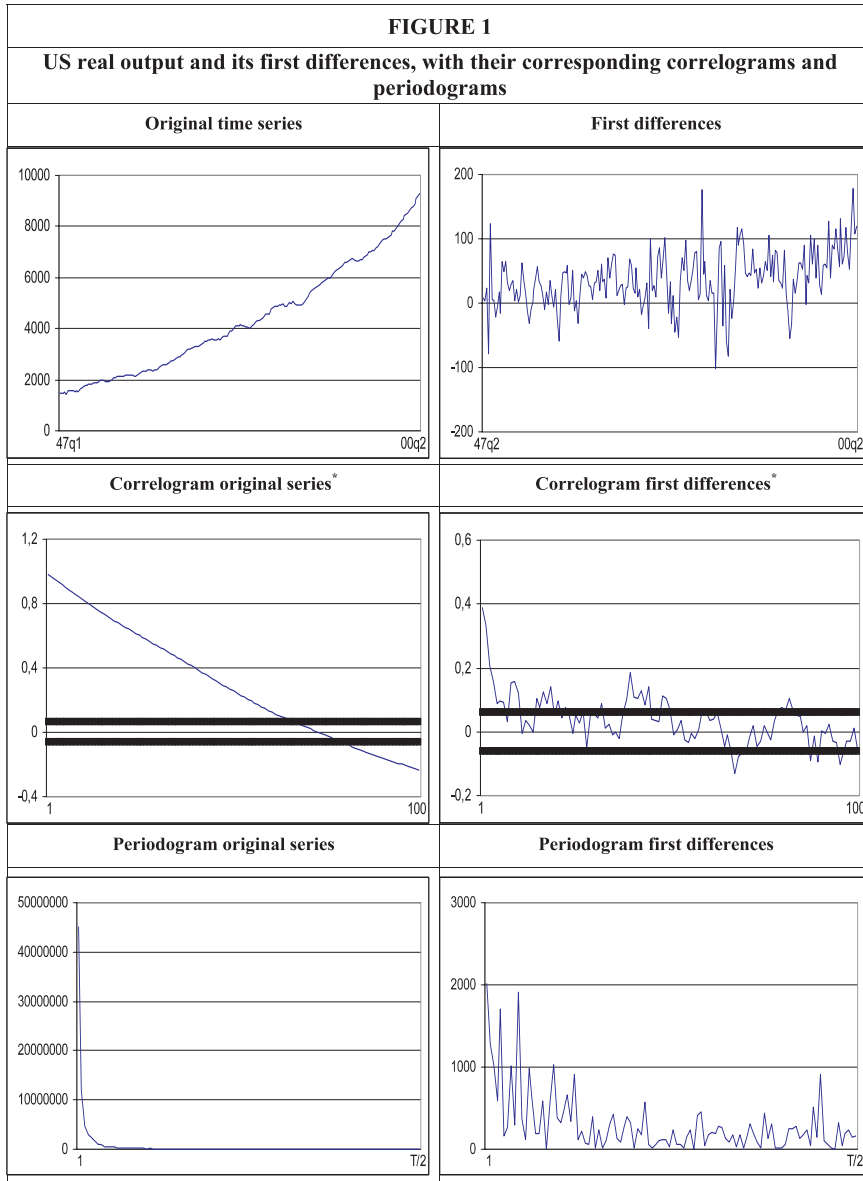
<sup>3</sup> These conditions are very mild regarding technical assumptions that are satisfied by the model in (7) – (9).

$$\hat{R} \rightarrow_d \chi_1^2, \quad \text{as } n \rightarrow \infty, \quad (15)$$

Thus, we are in a classical large sample testing situation by reasons described in Robinson (1994). Because  $\hat{R}$  involves a ratio of quadratic forms, its exact null distribution can be calculated under Gaussianity via Imhof's algorithm. However, a simple test is approximately valid under much wider distributional assumptions. Thus, an approximate one-sided 100 $\alpha$ %- level test of  $H_0$  (10) against the alternative:  $H_a: \theta > 0$  ( $\theta < 0$ ) will reject  $H_0$  if  $\hat{R} > z_\alpha$  ( $\hat{R} < -z_\alpha$ ), where the probability that a standard normal variate exceeds  $z_\alpha$  is  $\alpha$ . Furthermore, he shows that the above test is efficient in the Pitman sense, i.e., that against local alternatives of form:  $H_a: \theta = \delta n^{-1/2}$ , with  $\delta \neq 0$ , the limit distribution is normal with variance 1 and mean that cannot (when  $u_t$  is Gaussian) be exceeded in absolute value by that of any rival regular statistic. This version of Robinson's (1994) tests was examined in Gil-Alana (2001), and its performance in the context of unit root cycles was compared with Ahtola and Tiao's (1987) tests, the results showing that Robinson's (1994) tests outperform Ahtola and Tiao (1987) in a number of cases. Other versions of his tests have been applied to raw time series in Gil-Alana and Robinson (1997, 2001), testing for I(d) processes with the roots occurring respectively at zero and the seasonal frequencies. However, testing fractional cyclical models with the tests of Robinson (1994), this is the first empirical application, and one by-product of this work is its emergence as a credible alternative to the usual ARIMA (ARFIMA) specifications, which have become conventional in parametric modelling of macroeconomic time series.

### 3. The cyclical structure of the US real output

The time series data analysed in this section correspond to the quarterly, seasonally adjusted, real GDP in the US from 1947q1 to 2000q2, obtained from the Reserve Federal Bank of St. Louis' database. Alternatively, we could have employed seasonally unadjusted data, the reason for using adjusted data being that this series has been examined by many authors (e.g., Campbell and Mankiw, 1987; Diebold and Rudebusch, 1989; Hauser et al., 1992; Sowell, 1992a, and Koop et al., 2002). However, in all these papers they just concentrate at the long run or zero frequency and do not pay any attention to the cyclical structure underlying the series.



\* The large sample standard error under the null hypothesis of no autocorrelation is  $1/\sqrt{T}$  or roughly 0.06.

Figure 1 contains plots of the original series and its first differences, along with their corresponding correlograms and periodograms. We see that the original series increases over the sample period, though we also observe some apparent cyclical component in its behaviour. The nonstationary component of the series is also substantiated by the correlogram (with values de-



caying very slowly) and the periodogram (with a large peak around the smallest frequency). Taking first differences, the series may have a stationary appearance, though we still observe in the correlogram significant values even at some lags relatively away from zero, along with some apparent cyclical oscillation, which might imply the existence of some cyclical structure in the first differenced data.

The analysis below was first carried out based on the original data and there was not found any single evidence of fractional cycles across the sample. This is not surprising if we look at the periodogram of the original data, observing a large peak around the smallest frequency, which may suggest that first differences might be more appropriate when conducting this type of analysis<sup>4</sup>. In that respect, the model presented below might be related to the unobservable component (UC) models, (see, e.g., Harvey and Jaeger, 1993), where the long run component is specified as a random walk (or more generally,  $I(1)$ ) trend<sup>5</sup>.

We also conducted the analysis based on the log-transformed data. Note that the log-transformation renders numerous statistical and interpretation advantages. Thus, its first differences can be interpretable as the GDP growth and it removes the exponential trend in the original data. The results using the log-transformation were less persistent (with smaller orders of integration) than the ones presented below. However, we have preferred to work with the original data since it is still unclear how the order of integration is affected in the context of Gegenbauer processes by this type of transformation<sup>6</sup>.

Denoting the differenced series by  $y_t$ , we employ throughout model (7), (8) and (12), with  $z_t = (1, t)'$ ,  $t \geq 1$ ,  $(0, 0)'$  otherwise, i.e.,

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<sup>4</sup> In fact, several tests for unit roots and  $I(0)$  stationarity (Dickey and Fuller, 1979; Kwiatkowski et al., 1992; and Hobijn et al, 1998) were conducted on the original data and the evidence was in all cases in favour of unit roots.

<sup>5</sup> Similarly, we can relate this model with the Beveridge and Nelson's (1981) decomposition, which suggests, that any time series, which exhibits some form of nonstationarity of the type economic time series display, can be decomposed in two parts, a stationary component (the cyclical part) and a nonstationary one (the permanent component).

<sup>6</sup> Another argument that may be presented in favour of the log-transformed series is that whilst the first differences show volatility that is increasing over time, the log-differenced series shows a decline in volatility that has been subject in the recent empirical work (Kim and Nelson, 1999; McConnell and Perez-Quiros, 2000; Blanchard and Simon, 2001; Stock and Watson, 2002; etc.). However, the issue of volatility is out of the scope of the present work. Moreover, Robinson's (1994) procedure used in the paper is supposed to be valid in the context of non-Gaussian and heteroscedastic disturbances.

$$y_t = \beta_0 + \beta_1 t + x_t, \quad t = 1, 2, \dots \quad (16)$$

$$(1 - 2 \cos w_j L + L^2)^{d+\theta} x_t = u_t, \quad t = 1, 2, \dots, \quad (17)$$

testing  $H_0$  (10) for values  $d = 0, (0.10), 2$ , and  $w_j = 2\pi/j$ ,  $j = 2, \dots, n/2$ . Thus,  $j$  will indicate the number of periods required to complete the whole cycle. In other words, we test the order of integration of the cyclical polynomial at each of the frequencies of the process. Initially, we assume that  $\beta_0 = \beta_1 = 0$  a priori, i.e., we do not consider deterministic regressors in the undifferenced regression (16). Then, we also examine the cases of  $\beta_0$  unknown and  $\beta_1 = 0$  a priori, (i.e., with an intercept), and both  $\beta_0$  and  $\beta_1$  unknown (i.e., with an intercept and a linear time trend), and report the results based on white noise and autocorrelated disturbances<sup>7</sup>.

The test statistic reported across Table 1 corresponds to the one-sided statistic  $\hat{r}$  given by (14), for the case of white noise disturbances. However, instead of reporting the values for all possible combinations of  $d$  and  $j$ , we just report across the table, those cases where  $H_0$  cannot be rejected at the 95% significance level. We observe that the only values of  $d$  where  $H_0$  cannot be rejected correspond to  $d = 0.10$  and  $0.20$ , with  $j$  constrained between 7 and 24 periods. The results are very similar for the three cases of no regressors, an intercept, and a linear time trend, and the fact that  $H_0$  is rejected for all  $d$ , if  $j$  is smaller than 7 or higher than 24, along with all  $j$ , if  $d = 0$  or  $d \geq 0.30$ , suggests that the optimal local power properties of the tests, shown by Robinson (1994), may be supported by reasonable performance against non-local alternatives. Another remarkable feature observed across this table is that for a given frequency  $j$ , the value of the test statistic monotonically decreases as  $d$  increases. Thus, for example, if  $j = 7$ , the values of  $\hat{r}$  with  $d = 0.10$  are  $-0.305, -0.448$  and  $1.137$  respectively for the cases of no regressors, an intercept, and a linear trend, and testing  $H_0$ , for the same  $j$ , with  $d = 0.20$ , the values reduce to  $-2.097, -1.971$  and  $-1.561$ . This monotonicity is a characteristic of any reasonable statistic, given correct specification and adequate sample size. Thus, if  $H_0$  (10) is rejected for a given  $d$  (and  $j$ ) against alternatives of form:  $\theta < 0$ , an even more significant result in this direction should be expected when a higher value of  $d$  is tested. However, monotonicity is by no means necessary evidence of correct specification.

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<sup>7</sup> The FORTRAN codes of the programs used in this application are available from the author upon request.

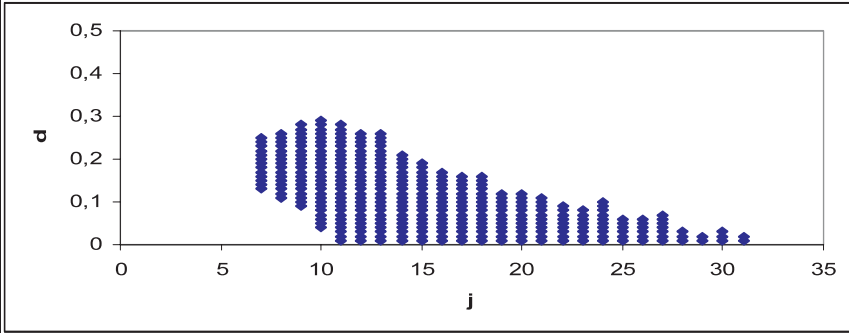
<b>TABLE 1</b>				
Non-rejection values of the null hypothesis at the 95% significance level, using white noise $u_t$				
j	d	With no regressors	With an intercept	With a linear trend
7	0.10	<b>-0.305</b>	<b>-0.448</b>	<b>1.137</b>
7	0.20	-2.097	-1.971	<b>-1.561</b>
8	0.10	<b>-0.413</b>	<b>-0.443</b>	<b>0.912</b>
8	0.20	-2.113	-1.879	<b>-1.471</b>
9	0.10	<b>1.351</b>	<b>1.260</b>	<b>0.584</b>
9	0.20	<b>-0.265</b>	<b>-0.285</b>	<b>-1.155</b>
10	0.10	<b>0.733</b>	<b>0.741</b>	<b>0.145</b>
10	0.20	<b>-0.516</b>	<b>-0.477</b>	<b>-1.242</b>
11	0.10	<b>0.327</b>	<b>0.345</b>	<b>-0.195</b>
11	0.20	<b>-0.780</b>	<b>-0.727</b>	<b>-1.438</b>
12	0.10	<b>-0.0002</b>	<b>0.024</b>	<b>-0.395</b>
12	0.20	<b>-1.013</b>	<b>-0.956</b>	<b>-1.546</b>
13	0.10	<b>-0.154</b>	<b>-0.173</b>	<b>-0.424</b>
13	0.20	<b>-1.078</b>	<b>-1.041</b>	<b>-1.484</b>
14	0.10	<b>-0.514</b>	<b>-0.489</b>	<b>-0.742</b>
14	0.20	<b>-1.522</b>	<b>-1.458</b>	<b>-2.113</b>
15	0.10	<b>-0.758</b>	<b>0.686</b>	<b>-0.905</b>
15	0.20	-1.998	<b>-1.584</b>	<b>-2.321</b>
16	0.10	<b>-0.953</b>	<b>-0.835</b>	<b>-1.018</b>
16	0.20	-2.115	<b>-1.740</b>	<b>-1.990</b>
17	0.10	<b>-1.056</b>	<b>-0.939</b>	<b>-1.095</b>
18	0.10	<b>-1.072</b>	<b>-0.991</b>	<b>-1.048</b>
19	0.10	<b>-1.401</b>	<b>-1.256</b>	<b>-1.358</b>
20	0.10	<b>-1.403</b>	<b>-1.252</b>	<b>-1.340</b>
21	0.10	<b>-1.546</b>	<b>-1.380</b>	<b>-1.454</b>
22	0.10	-1.765	<b>-1.528</b>	<b>-1.566</b>
23	0.10	-1.865	<b>-1.616</b>	<b>-1.637</b>
24	0.10	<b>-1.621</b>	<b>-1.423</b>	<b>-1.472</b>

In bold, the non-rejection values of the null hypothesis at the 95% significance level.

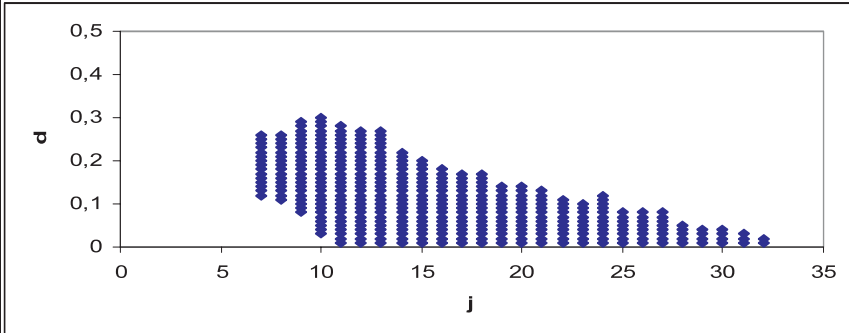
FIGURE 2

(d, j) combinations of the non-rejection values at the 95% significance level, with white noise  $u_t$ 

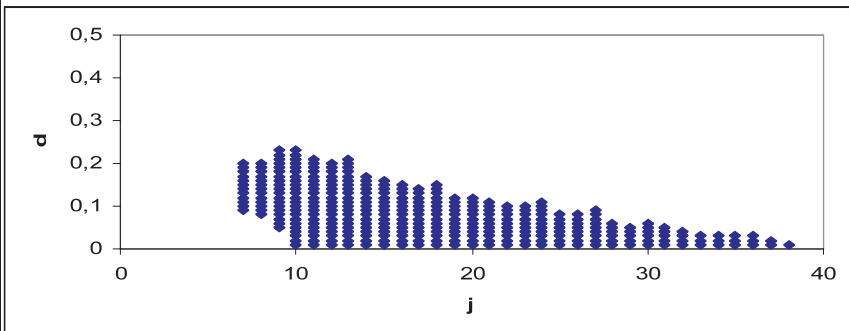
i): with no regressors



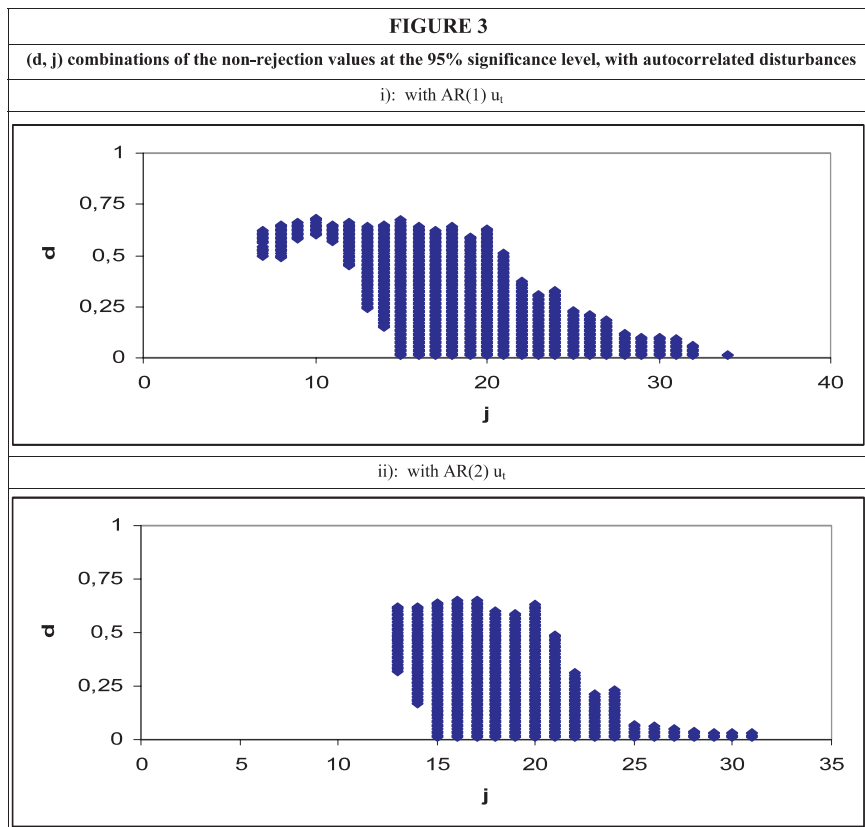
ii): with an intercept



iii): with a linear time trend



In order to have a more precise view about the non-rejection values obtained across Table 1, we have performed again the tests of Robinson (1994), but this time, for a range of values of  $d = 0, (0.01), 2$ . Figure 2 displays the  $(d, j)$  combinations where  $H_0$  cannot be rejected at the 95% significance



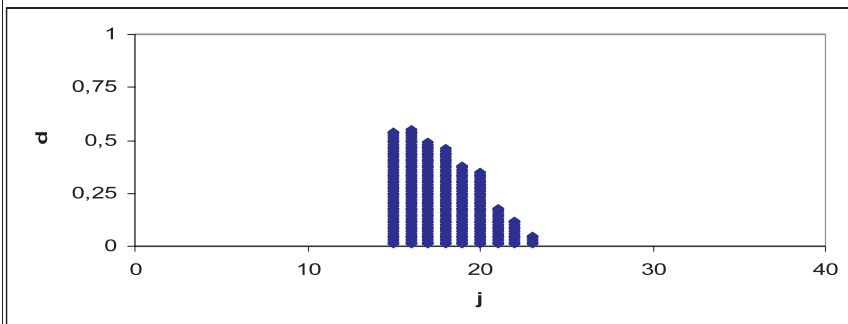
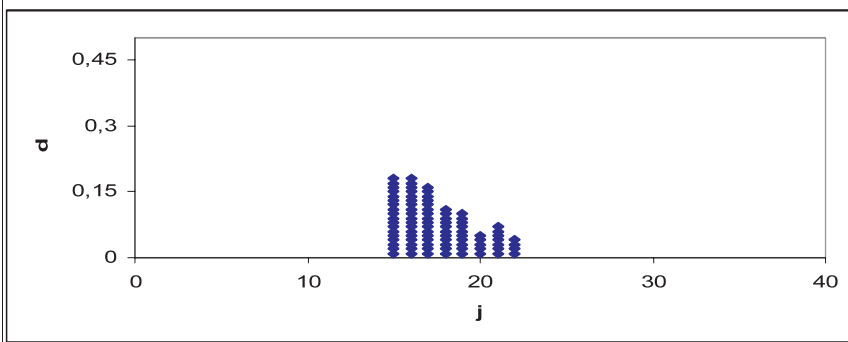
level. We see that the values of  $j$  range between 7 and 32, while  $d$  is in all cases smaller than 0.5, implying stationarity with respect to the cyclical component. These results are also in line with the literature mentioned in Section 1 about the duration of business cycles, with the periodicity constrained between 2 and 8 years.

The significance of the above results, however, might be in large part due to the un-accounted for  $I(0)$  autocorrelation in  $u_t$ . Thus, we also fitted other models, taking into account a weakly autocorrelated structure on the disturbances. We use AR(1) and AR(2) processes for  $u_t$  and the results, for the case of an intercept, are displayed in Figure 3. The first thing we observe here is that there is a larger proportion of non-rejection values compared with Figure 2 (white noise  $u_t$ ) and, though  $j$  is again constrained between 7 and 32 periods, the values of  $d$  widely range between 0 and 0.6.

The results presented so far are not much conclusive since we observe many non-rejections, depending on  $j$ ,  $d$ , the inclusion of deterministic trends and of the ARMA components. However, it should be noted that though the tests are unable to reject the null when  $d = 0$ , for some values of  $j$ , these hy-

FIGURE 4

(d, j) combinations of the non-rejection values at the 95% significance level, with autocorrelated disturbances and significant AR coefficients

i): with AR(1)  $u_t$ ii): with AR(2)  $u_t$ 

potheses are in all cases “less clearly non-rejected”<sup>8</sup> than when  $d$  is positive, suggesting that a component of long memory is present in relation with the cyclical structure.

Next, we try to be more specific about which may be the best model specification for this series, and the first thing we do is to examine the deterministic trends. We look at the coefficients  $\beta_0$  and  $\beta_1$  in (16) and, though not reported in the paper, the results showed that the coefficients corresponding to the time trend were insignificantly different from 0, in all cases where  $H_0$  could not be rejected. Note that these coefficients are all based on the null differenced model (13) and thus, standard t-tests apply. However, the coefficient for the case of an intercept was significant in many cases. Similarly for the weak dependence structure, we examine the coefficients corresponding to the AR components. Figure 4 resumes the (d, j) combinations of Figure 3 where all AR coefficients were significantly different from 0. We observe

<sup>8</sup> By ‘less clearly non-rejected’, we mean that the value of the test statistic is closer to the rejection critical values.

TABLE 2					
Selected models for the case of AR(1) $u_t$ with an intercept					
j	d	$\hat{r}$	$\beta_0$	$\tau_1$	Diagnostics
15	0.51	-0.004	34.330	-0.923	AC
16	0.37	-0.011	34.475	-0.917	AC
17	0.27	-0.013	34.589	-0.912	ABC
18	0.19	0.017	34.680	-0.906	ABC
19	0.14	0.020	34.754	-0.902	AC
20	0.12	0.014	34.816	-0.897	AC

A B and C means respectively that the models pass the diagnostic tests of no serial correlation (A), homoscedasticity (B) and functional form (C).

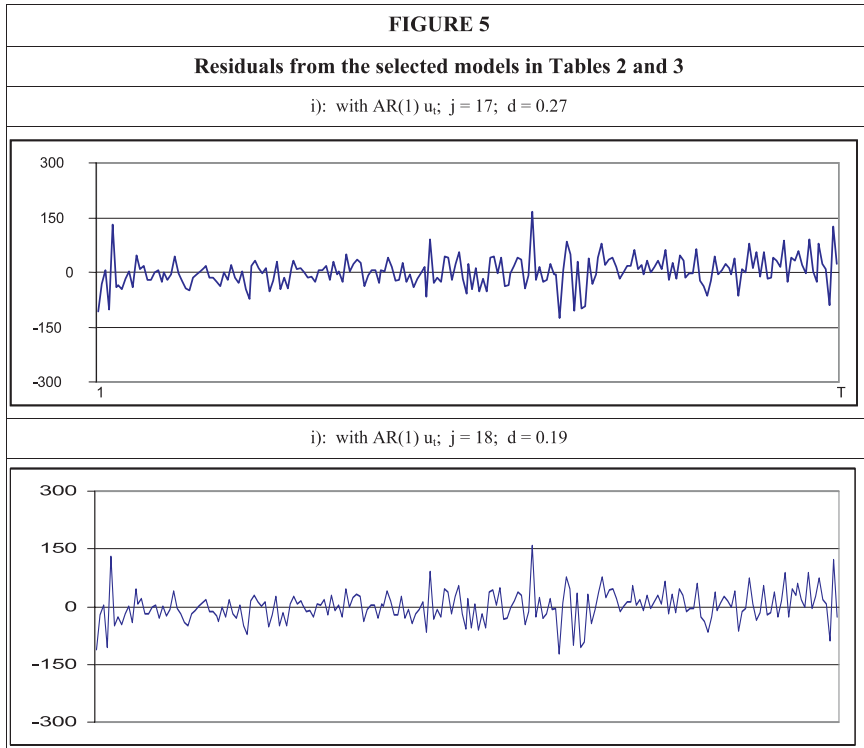
TABLE 3						
Selected models for the case of AR(2) $u_t$ with an intercept						
j	d	$\hat{r}$	$\beta_0$	$\tau_1$	$\tau_2$	Diagnostics
15	0.04	-0.058	34.404	-0.571	-0.296	A
16	0.04	-0.057	34.408	-0.404	-0.153	AB
17	0.02	-0.030	34.412	-0.251	-0.044	AB
18	0.02	0.063	34.414	-0.247	-0.037	A

A B and C means respectively that the models pass the diagnostic tests of no serial correlation (A), homoscedasticity (B) and functional form (C).

that the number of cases considerably reduces, especially for the AR(2) case: in both cases,  $j$  is constrained between 15 and 22, with  $d$  ranging between 0 and 0.50 for AR(1)  $u_t$  and between 0 and 0.20 for AR(2)  $u_t$ .

The results presented in Figures 3 and 4 correspond to the case of an intercept and AR(1) and AR(2)  $u_t$  respectively. Next, in Table 2 and 3, we choose for each  $j$ , the value of  $d$ , which produces the lowest  $|\hat{r}|$  across  $d$ . The intuition behind this is that, for each  $j$ , the model with the closest statistic to zero will also be the one with the closest residuals to white noise. The third column of the tables reports the values of the statistics. The next two columns (and three in case of AR(2)  $u_t$ ) correspond to the estimates of the intercept and the AR coefficients, while the last column reports some diagnostic tests carried out on the residuals of these selected models. An interesting feature observed across these tables is the fact that as we increase  $j$ , the value of  $d$  decreases, implying some type of competition between the length of the cycles and the orders of integration. The estimates of  $\beta_0$ ,  $\tau_1$  and  $\tau_2$  are significant in all cases.

Finally, we examine each of these potential models by looking at several diagnostic tests carried out on the residuals. In particular, we perform tests of no serial correlation, homoscedasticity and functional form, using Microfit.



Note that under the null hypothesis, the  $d_0$ -cyclical differences produce standard  $I(0)$  regressions (see equation (13)) and thus, standard techniques can be applied. The results are given in the last column of the tables. We see that the only models passing all the diagnostic tests on the residuals are those corresponding to AR(1)  $u_t$  with  $j = 17$  and 18. In the former case, the value of  $d$  is 0.27 while in the latter  $d = 0.19$ . Figure 5 displays the residuals for the two cases and we see that both have the appearance of white noise. Thus, we can conclude the analysis of this section by saying that the first differences of the US real output may be well described in terms of a Gegenbauer process, with the cycles occurring approximately every four years and a half, and with orders of integration ranging between 0.19 and 0.27, i.e., being stationary but with a component of long memory behaviour.

#### 4. A simulation study: Comparisons across models

The objective in this section is to compare the fractional cyclical models of Section 3 with other approaches based on ARIMA and ARFIMA models, in terms of their ability to reproduce the business cycle features. For this purpose, we need to describe a rule to date the business cycles and define their



characteristics. Numerous methods have been proposed in the literature. They can be based on direct data analysis (Burns and Mitchell, 1946), on expert judgement (NBER) or rely on the most recent econometric methods (Hamilton, 1994)<sup>9</sup>. In this paper, we have decided to consider exclusively classical cycles (directly extracted from the data in levels) in order to avoid statistical problems caused by the extraction of the cyclical component (See Canova, 1994). Besides, we apply the most common rule to date classical business cycles. This method was used in Candelon and Gil-Alana (2004), and it is at the basis of the famous program developed by Bry and Boschan (1971). It defines the phases of the business cycles as follows:

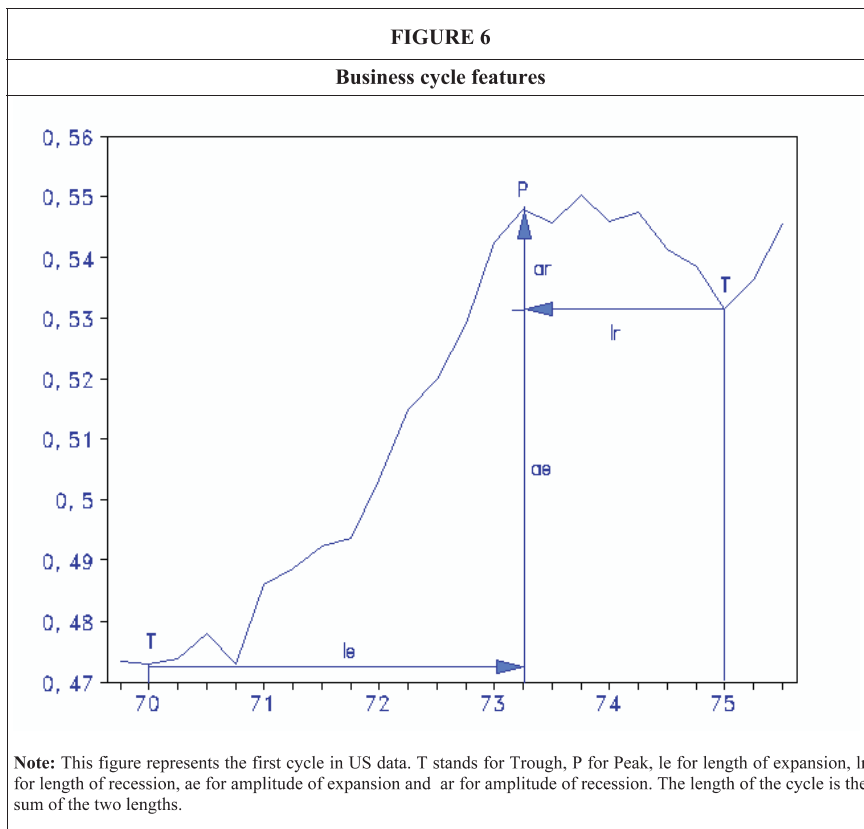
- a)  $y_{t-2} > y_{t-1} > y_t < y_{t+1}$ , then there is a trough at  $t$ , where  $y_t$  is, for example, the GDP in levels.
- b)  $y_{t-2} < y_{t-1} < y_t > y_{t+1}$ , then there is a peak at  $t$ .
- c) When several identical turning points are detected consecutively, we retain the optimal one (i.e., the highest peak and the deepest trough).

This rule is very intuitive because it simply considers that a turning point occurs after two consecutive periods of expansion and recession. Such a rule consists of detecting a change in the slope of the process: Conditions a) and b) can be rewritten as  $\Delta_j y_{t-j} > 0$  and  $\Delta_{j'} y_{t+j'} < 0$ , with  $j = 2$  and  $j' = 1$  for a peak (respectively as  $\Delta_j y_{t-j} < 0$  and  $\Delta_{j'} y_{t+j'} > 0$  for a trough). Such a definition insures that phases of the cycles have a minimum duration of 2 quarters and the completed cycles a minimum length of one year. This definition also presents the advantage of inducing an asymmetry in the length of the cycle phase, a property that is confirmed by historical data (Moore and Zarnowitz, 1982). On the contrary, we can not expect to detect an asymmetry in the amplitude of the phases, as the conditions on the change in slope are symmetric for troughs and peaks.

This method has been criticised in recent years. For example, it could exhibit not only major but also minor cycles. McNees (1991) and Webb (1991) propose to solve this problem via an increase in the reference period (for example, a peak could be characterised by 3 consecutive periods of growth over a year period). Candelon and Hénin (1995) have also noticed that this method leads to slight differences with the algorithms based on the detection of local optimum in the cases of growth cycles<sup>10</sup>. However, integrating these extensions in our dating algorithm will not alter the links between the degree of fractional cyclical integration and the business cycle

<sup>9</sup> See Pagan and Harding (1999) for an exhaustive survey of the procedures for determining turning points. Also, Harding and Pagan (2003).

<sup>10</sup> A local optimum is not a turning point for our methodology if it is preceded and followed by only one quarter of increase or decrease in the activity.



characteristics. We thus make the choice of simplicity and keep rules a) - c) as our dating algorithm.

From this dating, we have built five indicators (see Figure 6): the number of peaks (which corresponds to the number of cycles, as we consider that a cycle begins with a trough), the length of the cycles (period running between two successive troughs), the length and the amplitude of an expansion (period running from a trough to a peak), and the length and the amplitude of a recession (period running from a peak to a trough).

We have performed this dating algorithm for the series starting in 1960 in order to have better comparisons with reference studies. It is referred to the National Bureau of Economic Research (NBER) business cycle dating. It turns out in Table 4 that the algorithm leads to a nearly identical<sup>11</sup> dating except for the cycle (80:3-81:1), which is considered as minor in the official

<sup>11</sup> As NBER dating is performed for monthly data and our for quarterly observations, sometimes our results differ from a quarter.

TABLE 4					
Business cycle datation					
Country	Our datation			Reference datation*	
	Peak	Trough		Peak	Trough
UNITED STATES	73:2	70:1		73:4	70:4
	80:1	75:1		80:1	75:1
	81:1	80:3			
	90:2	82:1		90:3	82:4
	92:4	91:1			91:1

\* Reference datation corresponds to the NBER datation.

TABLE 5				
Business cycle characteristics of the US real GDP				
Number of peaks	Mean length of expansion	Mean length of recession	Mean amplitude of expansion	Mean amplitude of recession
5	21.25 (3.44)	5.00 (1.28)	0.16 (0.04)	0.02 (0.01)

Standard errors in parenthesis.

dating. In view of this, the above algorithm can be employed in simulated models when describing the business cycle characteristics.

Table 5 gathers the business cycle characteristics of the US real GDP. We notice that five major cycles occurred during the last 40 years. It also turns out that the expansions are longer and deeper than recessions. This stylised fact is generally known for classical cycles.

Table 6 reproduces the business cycle features of various models, which have been tried when modelling the US output. We start with AR(p)IMA(q) models with 1 and 2 differences. Here, we choose an ARIMA(1, 1, 1) and an ARIMA(1, 2, 1) respectively, and they have been chosen according to the Bayesian Information Criterion (BIC), from a group of sixteen potential models (with  $p$  and  $q \leq 3$ ). Besides, we consider fractional degrees of integration and thus, we also take into account an ARFIMA(0, 1.36, 0), which has also been selected by BIC<sup>12</sup>. and the ARFIMA(3, 0.41, 2) proposed by

<sup>12</sup> Here, we use the Sowell's (1992b) procedure of estimating by maximum likelihood ARFIMA(p,d,q) models with  $p$  and  $q \leq 3$ . Some theoretical properties of these stochastic processes can be found in Brockwell and Davies (1991), Odaki (1993) and Beran (1994). Moreover, when using integer differentiation we observe that if  $d = 1$ , the AR(1) coefficient is very large and close to 1, and, imposing  $d = 2$ , there is some evidence of overdifferentiation in the MA polynomial, implying that fractional processes with  $d$  lying between 1 and 2 (as in Sowell, 1992) might be more appropriate. Using the log-transformed data the persistence was found to be slightly smaller, which is in line with the results reported in Morley, Nelson and Zivot (2003) for the GDP growth rates.

TABLE 6				
Simulated business cycle characteristics of several models for the U.S. real GDP				
Model: ARIMA(1, 1, 1), $\phi = 0.95$ ; $\theta = -0.63$				
Aver. Number of Peaks	Mean length of expansion	Mean length of recession	Mean amplitude of expansion	Mean amplitude of recession
12 (0.045)	12.141 (3.654)	8.071 (2.321)	0.031 (0.010)	0.004 (0.043)
Model: ARIMA(1, 2, 1), $\phi = 0.30$ ; $\theta = -0.98$				
Aver. Number of Peaks	Mean length of expansion	Mean length of recession	Mean amplitude of expansion	Mean amplitude of recession
3 (0.035)	27.423 (4.215)	16.9742 (3.875)	.2502 (0.056)	0.1707 (0.051)
Model: ARFIMA(0, d, 0), $d = 1.36$				
Aver. Number of Peaks	Mean length of expansion	Mean length of recession	Mean amplitude of expansion	Mean amplitude of recession
9 (0.0761)	13.6392 (2.7195)	8.8280 (2.4950)	0.0994 (0.0101)	0.0237 (0.0089)
Model: ARFIMA(3, d, 2), $d = 0.41$ , $\phi_1 = -1.18$ ; $\phi_2 = 0.93$ ; $\phi_3 = -0.51$ ; $\theta_1 = -0.29$ ; $\theta_2 = 0.81$ (Sowell)				
Aver. Number of Peaks	Mean length of expansion	Mean length of recession	Mean amplitude of expansion	Mean amplitude of recession
14 (0.040)	10.008 (3.113)	6.143 (3.129)	0.123 (0.008)	0.023 (0.0001)
Model: (16) and (17) with AR(1) $u_t$ , $\beta_0 = 34.684$ ; $\beta_1 = 0$ ; $\tau = -0.906$ ; $j = 17$ ; $d = 0.27$				
Aver. Number of Peaks	Mean length of expansion	Mean length of recession	Mean amplitude of expansion	Mean amplitude of recession
5 (0.0564)	26.504 (3.123)	9.110 (3.452)	0.013 (0.006)	0.041 (0.034)
Model: (16) and (17) with AR(1) $u_t$ , $\beta_0 = 34.754$ ; $\beta_1 = 0$ ; $\tau = -0.902$ ; $j = 18$ ; $d = 0.19$				
Aver. Number of Peaks	Mean length of expansion	Mean length of recession	Mean amplitude of expansion	Mean amplitude of recession
5 (0.0487)	27.060 (3.190)	9.007 (2.543)	0.017 (0.007)	0.034 (0.027)

Sowell (1992b). Finally, the two selected models of Table 2, based on fractional cyclical integration, are also considered.

For each of these selected models, we simulate 2,500 series and compute their business cycle features. Their empirical means and variances are displayed in Table 6. Starting with the average number of peaks, we observe that the only models obtaining the exact number of peaks (5) are those based on the fractional cyclical structure. Also, the mean lengths of expansions and recessions are closer to the real business cycle characteristics with the latter models, though we lose part of the asymmetry in its behaviour. In relation with the amplitudes, the closest values to the true ones are obtained with the

I(d) model. As a conclusion, we can summarise the results obtained in this section by saying that the fractional cyclical models of Section 3 can be used as alternative credible ways when modelling real output, performing better than the classical ARIMA (and ARFIMA) models when reproducing the business cycle features of the data.

## **5. Concluding comments**

Economic theory in whatever format it comes has build-in the idea that cycles are stationary (if not in a wide sense, at least in a weak sense). That is, although there may be spillover of cyclical movements into the medium-long run, these are very small and nonstationary behaviour, if it exists, is due to the trend of the series. On the other hand, an enormous body of literature has also documented that a wide variety of indicators are persistent – where persistence is defined in many ways, but roughly speaking it means that the process has long memory. These two points are a datum and any empirical specification that contradicts these facts should be look with suspicion. In this article we present an empirical model that faces these two empirical facts.

We have examined the cyclical structure of the US real output by means of a procedure of Robinson (1994) that permits us to test unit and fractional root cycles in raw time series. This method is based on Gegenbauer processes and tests the degree of integration of the cyclical component at fixed frequencies over time, which permits us to approximate the length of the cycles. The tests have standard (normal) null and local limit distributions, which is another distinguishing feature of these tests compared with other procedures. Moreover, they do not require Gaussianity for the asymptotic distribution, with a moment condition only of order 2 required. The tests were performed based on the first differenced data and the results showed that fractional cycles may be an alternative plausible way of modelling this series, with the order of integration ranging between 0.19 and 0.27 and a periodicity constrained between 15 and 20 quarters of a year.

These results have strong implications in terms of economic policy and planning inference. Thus, shocks affecting the cyclical structure will be mean reverting (i.e. their effects disappearing in the long run), which is in contrast with the long run effects, where shocks will persist forever. Then, stronger policy actions will be required in relation with the long run effects in comparison with the cyclical ones, assuming, for example, that the trend and the cycle are driven by the same shock. It is important to note, however, that as it was concluded by the literature of the late eighties, trend-cycle decompositions are not unique and there is not clear matching between restrictions coming from economic theory and the restrictions needed to achieve statistical

identification. In that respect, the analysis of the forecasting ability of the proposed models is an area that should also be explored.

An argument that can be employed against the fractional cyclical model is that, contrary to seasonal cycles, business cycles are typically weak and irregular and are spread evenly over a range of frequencies rather than peaked at a specific value. However, contrary to that argument, we can explain that, in spite of the fixed frequencies used in this specification, the flexibility can be achieved throughout the first differencing polynomial, the ARMA components and the error term<sup>13</sup>. In that respect, the results presented here are completely in line with the literature on business cycle duration that says that cycles take place with a periodicity constrained between 3 and 8 years. Comparing these specifications with the classical ARIMA (and AR-FIMA) models, the results show that the former models better replicate the business cycle features of the US output. In fact, Hess and Isawa (1997) showed that the ARIMA models did it better than other linear and non-linear approaches (Perron, 1993; SETAR, Markov-Switching and Beaudry and Koop's 1993 non-linearity). Here, we have shown that fractional cyclical models can do it even better.

It would also be worthwhile proceeding to get point estimates for the fractional differencing parameter in the context of cyclical models. Some attempts have been made by Arteche and Robinson (2000) and Arteche (2002). However, the goal of this paper is to show that the Gegenbauer processes can be credible alternatives when modelling macroeconomic time series and, in that respect, the results presented in this paper lead us to unambiguous results, with the periodicity constrained between 15 and 20 periods and the order of integration higher than 0 but smaller than 0.5, implying stationarity and long memory in relation with the cyclical component.

The article can be extended in several other directions. First, the key results in the paper are based on first differenced data, and though the unit root null hypothesis is not rejected in the original series, it might also be of interest to perform other versions of Robinson's (1994) tests, which are larger in terms of model specification, for example, testing simultaneously the orders of integration at zero and the cyclical frequencies. However, in doing so, we lost the comparisons with other previous works, which are all based on first differences. Moreover, the existence of a unit root implies a stochastic trend and thus, the model can be alternatively written in the form of an orthogonal trend-cycle decomposition with an ARMA cycle which does not exhibit long memory. This specification is not nested in the model presented here, but it might be an alternative way of modelling its behaviour. Another potential

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<sup>13</sup> Bierens (2001) also use a model of this sort with  $d = 1$ , to test for the presence of business cycles in the annual change of monthly unemployment in the UK.

drawback of the present work is that it is based on an univariate model, with the limitation that it imposes in terms of theorizing, policy-making or forecasting. Theoretical models and policy-making involve the relationships between many variables and forecast performance can be improved through the use of many variables (e.g., factor based forecasts based on data involving hundreds of time series beat univariate forecasts, as described, for example, in Stock and Watson, 2002). However, the univariate work has relevance in the context of business cycles, firstly because different time series may have different amplitudes and different orders of integration, and there is not yet theoretical econometric models that permit us to examine cyclical fractional models in a multivariate framework. In that respect, the present paper can be considered as a preliminary step in the analysis of business cycles from a different time series perspective. Finally, the issue of data mining is another worry for economists when looking at time series models. There are so many possible models that may be relevant and so many modelling choices that econometricians are almost sure to find something purely by data mining. For this reason, sequential testing and other procedures based on information criteria are widely distrusted, and model averaging methods have become very popular. Thus, it might also be worthwhile to broaden the class of models under consideration and address the data mining problem, along with other issues (e.g., structural breaks) using averaging approaches. With respect to the latter issue, several papers conducted on the US output find little evidence of structural break in output, but strong evidence of structural break in output volatility, with the estimated break date around 1983. Diebold and Inoue (2001) point out that the evidence for long memory (at least at the zero frequency) in series that exhibit structural breaks may be spurious. We also conducted the analysis including a shift (and a slope) dummy for the break in 1983, and the results did not substantially differ from those reported across the paper.

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